Study regarding the implementation of an Ackerman steering geometry in MATLAB

D Moldovanu^{1*}, A Csato¹, N Bagameri¹

¹Technical University of Cluj-Napoca, Automotive Engineering and Transports Department, Bd. Muncii 103-105, Cluj-Napoca, Romania

dan.moldovanu@auto.utcluj.ro

Abstract. In dynamic vehicle behavior the steering system has an important role. The vehicle stability, maneuverability depends on steering system characteristics. Majority of passenger vehicles uses the rack pinion steering linkage system. In this article, a simplified planar six-bar linkage model of the rack pinion steering system is presented. Kinematic relations are used to determine every linkage position in every possible situation. This simplified system contains a rack, tie rods, steering arms, and wheels. The six-bar linkage model is introduced in MATLAB SIMULINK. In the function of the rack displacement, the wheel angels are determined. The dimensions of this planar model can be estimated due to the minimum error between the calculated wheel angels for the rack and pinion system and an Ackerman principle based system with similar dimensions. For optimizing a system and for verifying the method numerical calculation are used.

1. Introduction

The rack-and-pinion steering mechanism is widely used in passenger cars, majorly cars with independent front suspension. The rack and pinion steering mechanism has several advantages: simple construction, easy and cheap to manufacture, compact, and easy to handle [1]. The rack and pinion steering system consists of the rack, two tie rods and two steering arms of the wheel knuckle. Steering mechanisms are complex spatial linkages. Restrictions regarding on the positioning of the steering mechanism in the engine compartment and any significant kinematic correlation to the suspension, such as the cross-linking effect of steering and suspension, may further increase the complexity of describing the steering mechanism. Disregarding the dynamic effect of elasticity and a lateral slip of tires, the turning characteristics of the vehicle are remaining unaffected. The inclination angle of kingpin only slightly influences the transmission function of the steering, and it is chosen by other consideration as a vehicle moves straight forward provides stability of steering, not letting the wheels to get out from straight direction and provides a self-adjusting effect when cornering[2]. As a result the rack and pinion steering mechanism which is a spatial mechanism can be modelled as a planar six bar linkage system for verifying the Ackermann condition. Therefore, in this study the kingpin inclination angle and the caster angle are not taken into consideration and a six bar planar linkage system is considered as steering mechanism. The six bar linkage system consist of six revolute joints and one prismatic joint. Such a steering system has been used by other researcher [3], [4].

Another important characteristic of the vehicle is steady-state cornering. Vehicle at constant speed and constant steering angel making a circular motion within a circle with constant radius, this motion is called steady-state cornering. Vehicle motion characteristic can be described by vehicle steady-state cornering. For simplicity a single track bicycle model is used theoretical analysis of the vehicle cornering characteristics [5].

2. Objectives

In this research a rack and pinion steering mechanism with McPherson suspension is analysed. From the analysis of steering system of a projected six bar linkage mechanism with projected lengths are elaborated. By solving the nonlinear problem, a function is developed between the rack position and the wheel steering angle. A steering angel error is defined between the six bar linkage mechanism and the Ackerman condition. For minimizing the error a one parameter optimization problem is solved using the tie rod length as variable.

3. Ackerman condition

For the vehicle inner and outer wheel in steady-state cornering there is a kinematic relation called the Ackerman condition and it is defined by the equation (1) where δ_0 is the steer angle of the outer wheel, δ_i is the inner wheel steer angle, w is the track and l is the wheelbase of the vehicle. This condition means that the inner wheel has a bigger steer angle than the outer wheel, because is closer to the centre of rotation.

$$\cot\delta_0 - \cot\delta_i = \frac{w}{l} \tag{1}$$





4. The six bar linkage planar model

The planar model consist of the rack, two tie rods and two steering arms. The analytical method to d e

- s c r
- i

b

- e
- t
- h
- e
- r e
- 1
- ı a
- t t
- i
- 0



Figure 3. Half of the six bar linkage for solving the nonlinear equation.

$$\begin{cases} l_1 \sin\theta_1 + l_2 \sin\theta_2 - h = 0\\ l_1 \cos\theta_1 + l_2 \cos\theta_2 + \frac{s}{2} - x = 0 \end{cases}$$
(1)

$$l_1^2 - 2l_1(\sin\theta_1 + \cos\theta_1) + h^2 + l_2^2 - x^2$$
⁽²⁾

$$A = -2l_1s \tag{3}$$

$$B = -2l_1h \tag{4}$$

$$C = l_1^2 - l_2^2 + h^2 + s^2 \tag{5}$$

$$A\cos\theta_1 + B\sin\theta_1 + C = 0 \tag{6}$$

$$\cos\theta_1 = \frac{1-\tan^2\frac{\theta_1}{2}}{1+\tan^2\frac{\theta_1}{2}} \tag{7}$$

$$\sin\theta_1 = \frac{2\tan\frac{\theta_1}{1}}{1+\tan\frac{\theta_1}{2}} \tag{8}$$

$$\tan\frac{\theta_1}{2} = d \tag{9}$$

$$d^{2}(C - A) + 2Bd + C + A = 0$$
(10)

$$\Delta = 4B^2 - 4(C - A)(C + A)$$
(11)

$$d_{1,2} = \frac{-2B \pm \sqrt{\Delta}}{2(C-A)}$$
 (12)

5. Simulation model

For calculation of the wheel steering angels a MATLAB Simulink model was developed. The data for the projection lengths and angles of the different parameters of the six bar linkage mechanism are based on a VW Golf V short wheelbase model [7]. The parameters are presented in **Error! Reference source not found.**

Equations from section 4 are implemented in Simulink model for each wheel separately. Inside and the outside wheel are masked in separate blocks to get accurate results, but the major difference between the wheels is only the input, the rack position. The rack position movement is simulated with a signal builder which is linearly changing values from 0 to 80.



Figure 4. Simulink model of the six bar linkage.



Figure 5. Relations between inner and outer wheel angle for six bar linkage.



Figure 7. Relations between inner and outer wheel angle for Ackerman condition and for six bar linkage



Figure 6. Relations between rack position and wheel steer angle, δ_L for left (outer side) δ_R for right (inner side)



Figure 8. Sensitivity analysis for tie rod length.

6. Optimization

To have steering system working at very closely to the Ackerman condition the tie bar length is modified until de error between the two systems achieve the minimum. The error is calculated on each points of the two curves summing up this values in the sum of errors. For optimizing the six bar linkage a nonlinear program solver is used 'fmincon' which uses Hessian matrix for minimizing the cost function in this case the sum of errors.

After 18 iteration the algorithm has converge to the value of 122.5 mm of the tie rod and 3.3 for sum of errors (Figure 8). The optimized steering mechanism behaviour is represented in figure 9.





7. Conclusion

The purpose of the paper was to show the modelling of the Ackerman condition as a simplified model with a six-bar linkage. By using MATLAB Simulink the calculations were made and optimized by using 'fmincon' which uses Hessian matrix for minimizing the cost function in this case the sum of errors, which has brought the relations between inner and outer wheel angle for Ackerman condition very close. Further simulations will include the control of the steering as a closed loop system.

References

- Wasiwitono, U., Sidarta, I., Sigit Pramono, A., Sutikno, S., Wikarta, A., 'Steering System Kinematic and Steady-State Cornering Analyses of the ITS Electric Car', *IPTEK J. Proc. Ser.*, 2014.
- [2] Mazilu, A., Preda, I., 'Kinematic Optimization of the Rack and Pinion Steering-System of an Automobile: An Example', 2016.
- [3] Habibi, H., Shirazi, K. H., Shishesaz, M., 'Roll steer minimization of McPherson-strut suspension system using genetic algorithm method', *Mech. Mach. Theory*, vol. 43, no. 1, pp. 57–67, 2008.
- [4] Rahmani, A., Rao, P. V. M., Saha, S., 'Kinematic and sensitivity analysis and optimization of planar rack-and-pinion steering linkages', *Mech. Mach. Theory - MECH MACH THEOR*, vol. 44, pp. 42–56, 2009.
- [5] Yang, S., Lu, Y., Li, S., 'An overview on vehicle dynamics', *Int. J. Dyn. Control*, vol. 1, no. 4, pp. 385–395, Dec. 2013.
- [6] Jazar, R., Vehicle Dynamics: Theory and Application. 2008.
- [7] Knapczyk, J., Kucybała, P., 'Simplified planar model of a car steering system with rack and pinion and McPherson suspension', *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 148, p. 12011, 2016.
- [8] Antonya Cs, Butnariu S and Beles H 2015 Geometric identification of a four-bar linkage from noisy tracking data. 2015 IFToMM World Congress Proceedings, IFToMM 2015, Pages: 217-222. 14th; Taipei; Taiwan; 25 - 30 October 2015; Code 127160. Publisher: National Taiwan University, DOI: 10.6567/IFToMM.14TH.WC.OS2.012.